where T_{0b} is the bulk temperature at the entrance. If the fluid enters the duct at constant temperature, T_0, then $T_{0b} = T_0$.

By combining equations (8), (12) and (15) it is found that

$$B = T_{0b} - \frac{q_w}{2kr_h}\overline{r^2}. \tag{16}$$

Thus one can determine the temperature of the fluid at any location past the thermal entrance region by knowing only the temperature of the fluid at the duct's entrance. If on the other hand the temperature boundary condition is given by equation (3b) instead of (3a)

$$B = T_1 - \frac{q_w}{2kr_h}\left(r_1^2 + \frac{4\alpha z_1}{u}\right). \tag{17}$$

The square of the radius of gyration, $\overline{r^2}$, can be found by integrating over each region A_i and A_j as shown in Fig. 2. The results add to give

$$\overline{r^2} = \frac{r_h^2}{2} + \frac{1}{6P}\sum_{i=1}^{m}\left[e_i^3 + (L_i - e_i)^3\right] \tag{18}$$

for any inscribable duct. Also for any triangular duct

$$\overline{r^2} = \frac{r_h^2}{3} + \frac{ab + ac + bc}{6} - \frac{4}{3}\frac{abc}{P} \tag{19}$$

where a, b and c are the lengths of the three sides. And for any m sided regular polygonal duct

$$\overline{r^2} = \frac{r_h^2}{2}\left(1 + \frac{1}{3}\tan^2\frac{\pi}{m}\right). \tag{20}$$

A summary of these results is given in Table 1.

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Int. J. Heat Mass Transfer. Vol. 12, pp. 1191–1195. Pergamon Press 1969. Printed in Great Britain

RADIATION EFFECT ON THE ENTHALPY AND VELOCITY DISTRIBUTIONS OF A LAMINAR, COMPRESSIBLE, PLANAR FREE JET

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(*Received* 28 *August* 1968 *and in revised form* 3 *February* 1969)

NOMENCLATURE

- a_i, functions defined by equations (26);
- b_i, functions defined by equations (26);
- C_p, specific heat at constant pressure;
- F, velocity function;
- G, a constant defined by equation (10b);
- G_i, functions governed by equations (21);
- H, total heat released at the jet entrance;
- h, specific enthalpy;
- h^*, specific enthalpy of the reference state;
- K, thermal conductivity;
- K_p, Planck mean absorption coefficient;
- L, radiation loss parameter defined by equation (9);
- M, total momentum released at the jet entrance;
- m, a function defined by equation (15);
- n, radiation loss parameter defined by equation (9);
- P_r, Prandtl number;
- Q_r, radiation loss per unit mass;
- q_i, functions defined by equations (28);
- r_i, functions defined by equations (28);
- S, a similarity variable defined by equation(10a);
- T, temperature;
- U_o axial velocity given by equation (15);
- u, v, velocity components in Cartesian system;
- x, y, spatial coordinates;
- Z_i, functions defined by equation (27).

Greek symbols

- Γ, a function defined by equation (18);
- η, a smilarity variable defined by equation (10a);
- μ, viscosity;
- ρ, density;
- ρ^*, density of the reference state;
- σ, Stefan-Boltzmann constant.

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Subscripts

i, designate the zero-, first-, and second-order radiation effects, respectively, $= 0, 1, 2$.

INTRODUCTION

THE study of the momentum and energy fields in free jet flows has received considerable attention in the past and the present [1, 2]. It is well known that in free jets associated with heat release, the determination of the velocity and the temperature distributions requires the solution of the coupled continuity, momentum, and energy equations with the equation of state. In considering the energy equation, it has been usual to disregard the possibility that the flowing gas could be radiating although this may alter the temperature and the velocity distributions in the jet. Recently, some authors have investigated the effect of radiation transport on the flow and energy in free jets [3, 4]. However, very few quantitative results have been reported to weigh the relative effect of radiation to convection transport in the jet.

The present paper deals with the momentum and energy transport in a radiating, laminar, compressible, planar free jet. The flow model considered here is a jet issuing into a quiescent atmosphere chosen to be in a thermodynamic reference state (25°C and 1 atm pressure). The radiative process is described by the optically thin model. The gas is usually considered to be optically thin when the characteristic length of the free jet is small or the radiating gas is diluted by other non-radiating gases, as in many cases of practical applications. In this analysis, the governing equations are investigated by a boundary layer type of analysis. The radiation loss term occurring in the energy equation was found to be expressed by a power function of the enthalpy for most radiating gases up to a temperature of 5,000°R. This is different from the grey gas analysis in that the temperature dependence of the absorption coefficient is taken into account here. Based on the power law model, it is shown that the effect of radiation on the enthalpy distribution could be treated as a perturbation on the zero-radiation solution, and the perturbation functions are given by linear second-order ordinary differential equations. Explicit approximate expressions for the perturbation functions are obtained by using integral methods. The numerical calculations show that the specific enthalpy of a radiating gas decreases from its zero-radiation value at any point in the jet. Consequently the density increases, and as a result of momentum conservation the velocity decreases from its zero-radiation value except at the center of the jet.

ANALYSIS

Basic equations

The governing equations for a radiating, laminar, compressible, planar free jet subject to the usual boundary layer approximations are,

$$(\rho u)_x + (\rho v)_y = 0 \tag{1}$$

$$\rho u u_x + \rho v u_y = (\mu u_y)_y \tag{2}$$

$$\rho u h_x + \rho v h_y = \left(\frac{\mu}{Pr} h_y\right)_y - \rho Q_r \tag{3}$$

$$\rho/\rho^* = h^*/h \tag{4}$$

where the density ρ, the enthalpy h and the velocity components u and v are functions of the spatial coordinates x and y. The density ρ^* and the enthalpy h^* are those of a suitable reference state, Q_r is the radiation loss per unit mass.

With the surrounding atmosphere chosen to be at rest and at a thermodynamic reference state (25°C and 1 atm pressure) for simplification of the analysis, the boundary conditions on equations (1)–(3) are given by:

$$v = 0, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial h}{\partial y} = 0 \quad \text{at } y = 0 \tag{5a}$$

$$u = 0, \quad h = 0 \quad \text{at } y = \infty. \tag{5b}$$

Since the pressure is constant and the surrounding atmosphere is non-radiating, the velocity and temperature distribution must satisfy the conditions of momentum and energy conservation, i.e.

$$M = \text{constant} = 2 \int_0^\infty \rho u^2 \, dy \tag{6}$$

and

$$H = \text{constant} = 2 \int_0^\infty \rho u h \, dy + 2 \int_0^x \int_0^\infty \rho Q_r \, dy \, dx. \tag{7}$$

Equations (1)–(7) give complete specification of the problem. Before starting with the analysis, the functional dependence of the radiation loss on the enthalpy will be determined as follows: The radiation loss term in equation (3) is often written in terms of the Planck mean absorption coefficient as,

$$Q_r = 4\sigma K_p T^4 \tag{8}$$

where K_p is the Planck mean absorption coefficient. For radiating gases up to a temperature of 5,000°R, values of K_p have been calculated from spectroscopic data in [5] and [6]. Using the thermodynamic tables on temperature–enthalpy transformation [7] and the calculated values of K_p, the radiation loss Q_r given by equation (8) was plotted vs. the enthalpy as shown in Fig. 1. It was found that Q_r could be easily fitted by an equation of the type;

$$Q_r = L\, h^n \tag{9}$$

where the power term n varies approximately between 1·75 and 2·0.

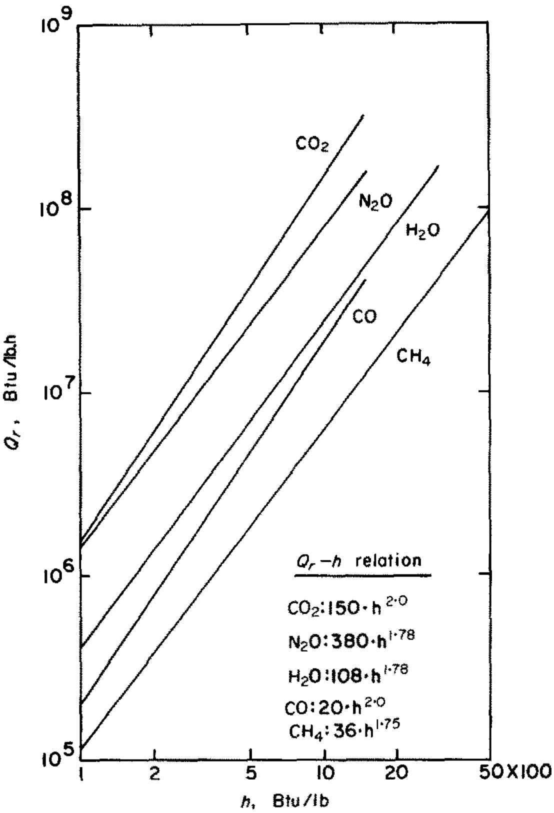

FIG. 1. Radiation energy loss per unit mass Q_r as function of the enthalpy h at a pressure of one atmosphere.

The flow problem

It can be shown [8] that, using the following similarity variables

$$S = (2\int_0^x \rho^*\mu^* U_0\, dx)^{1/2}, \qquad \eta = \frac{U_0}{S}\int_0^y \rho\, dy \tag{10a}$$

$$F'(\eta) = u/U_o, \qquad G = \rho\mu/(\rho^*\mu^*) = \text{const.},$$

$$P_r = \mu C_p/K = \text{const.} \tag{10b}$$

the governing equations (2) and (6) for the velocity distribution reduce to

$$GF''' + FF'' + (F')^2 = 0 \tag{11}$$

$$M = 2U_0 S\int_0^\infty (F')^2\, d\eta \tag{12}$$

subject to the boundary conditions

$$F(0) = F''(0) = 0, F'(\infty) = 0. \tag{13}$$

Equation (11) with its boundary conditions given by equation (13) possess an exact solution [1, 9],

$$F' = \text{sech}^2\, [\eta/\surd(2G)]. \tag{14}$$

Using equations (12) and (14), the axial velocity U_0 could be determined as,

$$U_0 = \frac{m}{S} \qquad \text{where } m = 3M/[4\surd(2G)]. \tag{15}$$

The thermal problem

By using the similarity variables given by equations (10), the governing equations (3) and (7) for the energy distribution [with Q_r given by equation (9)] reduce to,

$$\frac{G}{P_r}h'' + (Fh)' - F'\frac{\partial(Sh)}{\partial S} = \Gamma S^4 h^n \tag{16}$$

$$H = 2S\int_0^\infty F'h\, d\eta + 2\Gamma\int_0^S\int_0^\infty S^4 h^n\, d\eta\, dS \tag{17}$$

where Γ is given by:

$$\Gamma = 32GL/(9M^2\rho^*\mu^*). \tag{18}$$

Equation (16) is subject to the boundary conditions:

$$h'(0) = 0, \qquad h(\infty) = 0. \tag{19}$$

It is known [9] from the zero-radiation solution of equation (16) that the enthalpy $h = 0[1]/S$. Therefore under the condition of $0 < n < 5$ and with $S \leqslant 0$ [1], the radiation effect which is represented by the right-hand side of equation (16) on the enthalpy distribution can be considered as a perturbation effect on the zero-radiation solution provided that $\Gamma \ll 1$. Since most radiating gases in the temperature range of the present study are included in the range of $0 < n < 5$, a solution of equation (16) which satisfies equation (17) can be written as:

$$h(S, \eta) = G_0(\eta)/S + \Gamma S^{4-n}\, G_1(\eta) + \Gamma^2 S^{9-2n}\, G_2(\eta) + \tag{20}$$

substituting this expression in equation (16) and recognizing that the order of magnitude of the second and third terms in equation (20) is much less than that of the first term, after equating the coefficients of ΓS^r on both sides of equation (16), one arrives at the following set of equations:

$$\frac{G}{P_r}\, G_0'' + (FG_0)' = 0 \tag{21a}$$

$$\frac{G}{P_r}\, G_1'' + (FG_1)' - (5-n)F'G_1 = G_0^n \tag{21b}$$

$$\frac{G}{P_r}\, G_2'' + (FG_2)' - (10-2n)F'G_2 = nG_0^{n-1}G_1 \tag{21c}$$

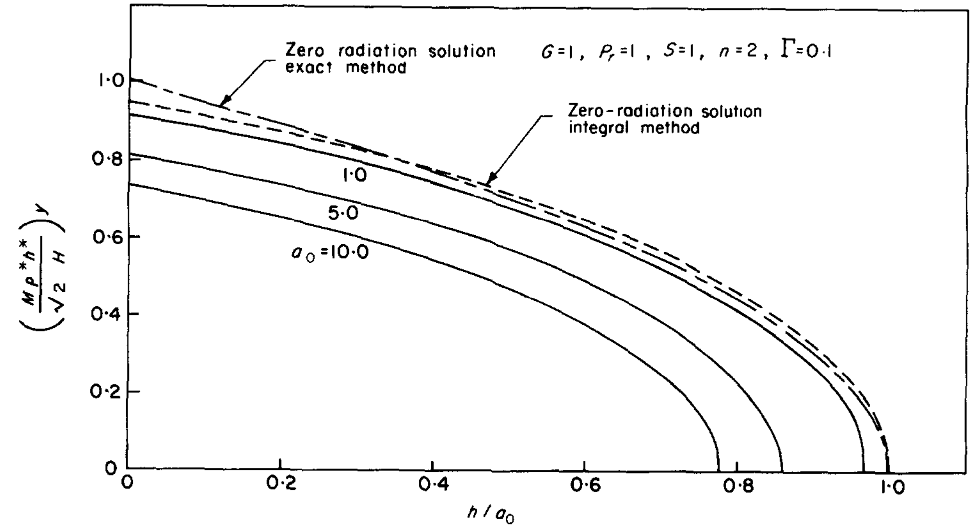

FIG. 2. Radiation effect on the enthalpy distribution for different values of heat release.

subject to boundary conditions:

$$G_i'(0) = 0 \quad \text{and} \quad G_i(\infty) = 0 \quad \text{for } i = 0, 1, 2 \tag{22}$$

while equation (17) takes the form:

$$H = 2\int_0^\infty F'G_0\,d\eta + 2\Gamma S^{5-n}\left[\int_0^\infty F'G_1\,d\eta + \frac{1}{(5-n)}\int_0^\infty G_0^n\,d\eta\right] + 2\Gamma^2 S^{10-2n}\left[\int_0^\infty F'G_2\,d\eta + \frac{n}{(10-2n)}\int_0^\infty G_0^{n-1}G_1\,d\eta\right] + \ldots \tag{23}$$

by use of equations (21) and the boundary conditions given by equation (22), it can be easily shown that the expressions in parenthesis cancel.

Equations (21) form a set of linear second-order ordinary differential equations which can be easily solved in sequence by numerical computations. It can be recognized that equation (21a) is the governing equation for the zero-radiation solution which was found to possess an explicit solution [9]. By using the solution of equation (21a), subsequent solutions for the perturbation functions G_1 and G_2 could be obtained for any radiating gas once its value of n is specified. An alternative procedure which renders an explicit, but approximate solution for the enthalpy h will be derived here. This method is based on the integral methods which has been previously applied in investigating the flow field in wakes and jets [10, 11].

The approximate solutions of equations (21) are obtained by assuming:

$$G_i = a_i \exp\left[-Prb_i\eta^2/(2G)\right] \quad \text{for } i = 0, 1, 2 \tag{24}$$

and

$$F' = \exp\left[-\eta^2/(2G)\right] \tag{25}$$

where a_i and b_i are constants and can be determined by satisfying both equation (21) at $\eta = 0$ and the constraints implied by equation (23). After solving the resulting algebraic equations, the values of a_i's and b_i's were found to be given by:

$$a_0 = \left(\frac{1 + Pr}{2\pi}\right)^{\frac{1}{2}} \cdot \frac{H}{\sqrt{G}}, \quad b_0 = 1 \tag{26a}$$

$$a_1 = -a_0^n Z_1/[(5-n)\sqrt{n}], \quad b_1 = Z_1^2 - 1/Pr \tag{26b}$$

$$a_2 = -na_0^{n-1}a_1 Z_2/\{(10-2n)\sqrt{[(n-1) + b_1\ ]}\}, \quad b_2 = Z_2^2 - 1/Pr \tag{26c}$$

where

$$Z_i = [r_i + (q_i^3 + r_i^2)^{\frac{1}{2}}]^{\frac{1}{3}} + [r_i - (q_i^3 + r_i^2)^{\frac{1}{2}}]^{\frac{1}{3}} \quad \text{for } i = 1, 2 \tag{27}$$

and

$$r_1 = \frac{(5-n)\sqrt{n}}{2}, \quad q_1 = \frac{(4 - n - 1/Pr)}{3} \tag{28a}$$

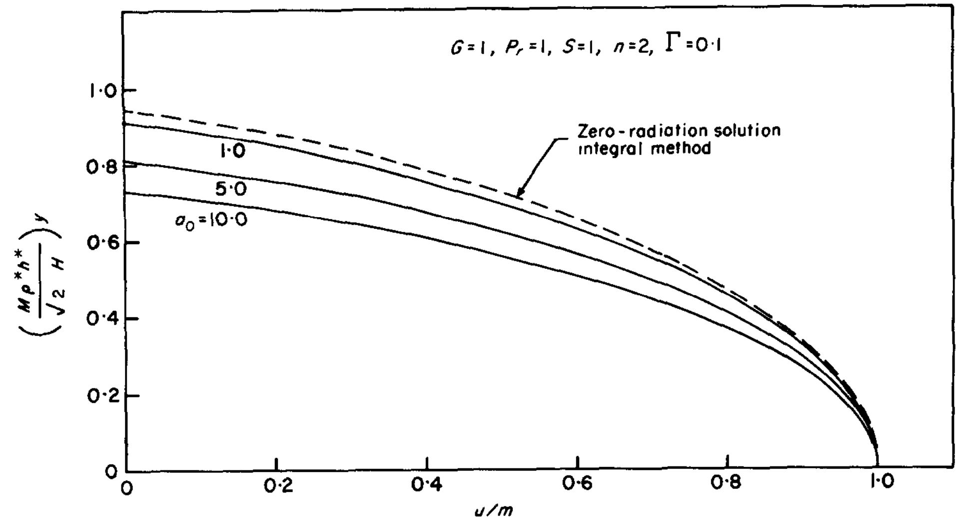

FIG. 3. Radiation effect on the velocity distribution for different values of heat release.

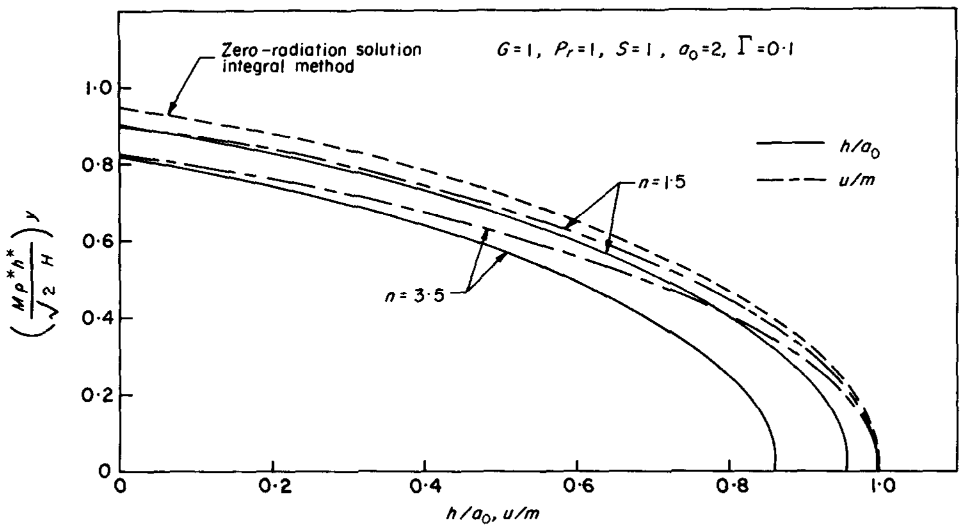

FIG. 4. The enthalpy and the velocity distributions for different values of the radiation loss.

$$r_2 = (5 - n)\sqrt{[(n-1) + b_1]},$$

$$q_2 = \frac{(9 - 2n - 1/Pr)}{3} \qquad (28b)$$

the nature of the expression for $Z_i (i = 1, 2)$ given by equation (27) arises from the fact that the a_i's are governed by third-order polynomial equations.

With the values of the G_i's given by equation (24), it is possible to determine explicitly the velocity and the enthalpy distributions in terms of the physical coordinates x and y. The transformation from the $S - \eta$ coordinates to the $x - y$ coordinates can be accomplished by considering equations (10a) and (15) for determining the coordinate x and equations (4), (10a), (15) and (20) for y. The result is:

$$x = \frac{4\sqrt{(2G)}}{9\mu^*\rho^*M} S^3 \qquad (29a)$$

and

$$y = \frac{4SG\sqrt{(\pi/P_r)}}{3\rho^* h^* M}\left\{a_0 \operatorname{erf}\left[\sqrt{\left(\frac{Pr}{2G}\right)}\,\eta\right] + \Gamma S^{5-n}\frac{a_1}{\sqrt{b_1}}\operatorname{erf}\left[\sqrt{\left(\frac{P_r b_1}{2G}\right)}\,\eta\right] + \Gamma^2 S^{10-2n}\frac{a_2}{\sqrt{b_2}}\operatorname{erf}\left[\sqrt{\left(\frac{P_r b_2}{2G}\right)}\,\eta\right] + \ldots\right\} \quad (29b)$$

RESULTS AND DISCUSSION

It is clear, from the nature of the radiation function in equation (3) that the effect of radiation is to decrease the enthalpy from its zero-radiation value at any point in the jet. Such decrease will depend on the heat released at the jet entrance and the functional behaviour of the radiation loss on h. Values of h as a function of the distance y from the jet axis are plotted in Fig. 2 by use of equations (20), (24) and (29b) for different values of heat release and for selected values of the other parameters. Shown also in the figure is the zero-radiation solution obtained by both exact and integral methods. It is clear that the integral methods yield a solution which is quite close to that obtained exactly. Therefore, the solution for the enthalpy with radiation obtained by using the integral methods could be considered reasonably accurate.

The decrease in the value of the enthalpy due to radiation will result in an increase in the local density as given by equation (4). Since the momentum at any plane normal to the jet axis is conserved, the increase in the density will result in a decrease in the velocity, except at the jet axis where the velocity is governed by the momentum release at the jet entrance. Representative values of the velocity distribution versus the coordinate y are shown in Fig. 3. Finally, the dependence of the velocity and the enthalpy distributions on the radiation loss function is shown in Fig. 4 for a fixed value of heat released at the jet entrance. It is clear from the figure that for large values of n, radiation plays an important role on the energy transport in free jets.

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Int. J. Heat Mass Transfer. Vol. 12, pp. 1196–1201. Pergamon Press 1969. Printed in Great Britain

AN EXPERIMENTAL INVESTIGATION OF THE INFLUENCE OF SLOT-LIP-THICKNESS ON THE IMPERVIOUS-WALL EFFECTIVENESS OF THE UNIFORM-DENSITY, TWO-DIMENSIONAL WALL JET

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(*Received* 1 *January* 1969)

NOMENCLATURE

a, constant;
b, constant;
m, mass concentration;
t, thickness of slot lip;
u, velocity in x-direction;
$\bar{u}_C$, average velocity through the slot

$$\equiv \frac{1}{y_C}\int_0^{y_C} u\,dy;$$